Application of Bifurcation Methods for the Prediction of Low-Speed Aircraft Ground Performance

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The design of aircraft for ground manoeuvres is an essential part in satisfying the demanding requirements of the aircraft operators. Extensive analysis is done to ensure that a new civil aircraft type will adhere to these requirements, where the nonlinear nature of the problem generally adds to the complexity of such calculations. Small perturbations in velocity, steering angle or brake application may lead to significant differences in the final turn-widths that can be achieved. Here, the U-turn manoeuvre is analysed in detail, with a comparison between the two ways in which this manoeuvre is conducted. A comparison is also made between existing turn-width prediction methods that consist mainly of geometric methods and simulations, and a proposed new method that uses dynamical systems theory. Some assumptions are made with regards to the transient behaviour, where it is shown that these assumptions are conservative when an upper bound is chosen for the transient distance. Furthermore, we demonstrate that the results from the dynamical systems analysis are sufficiently close to the results from simulations to be used as a valuable design tool. Overall, dynamical systems methods provide an order of magnitude increase in analysis speed and capability for the prediction of turn-widths on the ground, compared to simulations.

I. Introduction

The Boeing 747 has long been used as the baseline for specifying requirements that large international airports have to adhere to. This means that new civil aircraft designs have to stay within the manoeuvrability requirements of this aircraft, to ensure that no significant investment is needed for upgrades to existing airport infrastructure. This is also the case for the Airbus A380, for which only terminal facilities need to be upgraded, while runways and taxiways do not require any significant alterations. Considerable analysis is thus needed in the early design stages to guarantee the desired ground performance of a proposed new civil aircraft.

This paper indicates the types of ground manoeuvres that are of importance in the design of civil aircraft, and how they can be analysed efficiently. As a starting point we argue that there is still a lack of a global understanding of ground manoeuvres with the methods of analysis that are currently in use. We then introduce new methods from dynamical systems theory, and discuss how they can be employed to gain a complete characterisation of the aircraft's dynamic performance on the ground.

The most basic and widely used analysis techniques for early design use purely geometric and static relationships between the gear positions to determine the turn radius of the aircraft. However, it can be shown that the centre of gravity (CG) location, tyre and brake properties do in fact play a significant role in the overall performance, which means that the static gear layout alone does not determine the possible turn radius. As is shown, different types of ground manoeuvres can be analysed via the simulation of suitable aircraft models in multi-body simulation software, an approach that has become an additional tool for this

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type of analysis. We will specifically focus on the U-turn manoeuvre and analyse the way in which this manoeuvre is conducted. This focus is partly due to the fact that not all the parameters that define a U-turn manoeuvre are entirely understood, but also because this is one of the more demanding ground manoeuvres to perform.

One important property of the overall aircraft model lies in the nonlinear nature of landing gear components, for example, due to geometric effects or large tyre deflections. Therefore, small perturbations in velocity, steering angle or brake application may lead to significant differences in the final turn-width. This sensitivity means that a large number of simulation runs need to be conducted to ensure that all feasible scenarios have been taken into account. In a previous study we showed that it is possible to calculate the radius of turn for a specific thrust case as the steering angle is varied [1]. In this paper a comparison is made between the most widely used methods that consist of geometric and simulation approaches, and a new approach where dynamical systems theory is used to determine the turn-widths and feasible regions for specific ground manoeuvres.

More specifically, the method of numerical continuation, as implemented in the software package AUTO [2], allows one to construct bifurcation diagrams as functions of one or more operational and/or design parameters [1, 3–5]. To allow for the analysis of industrially-tested models in a user-friendly environment, AUTO has been integrated with MATLAB in the form of a Dynamical Systems Toolbox. A SimMechanics [6] model of the A380 was developed and validated against existing models and flight test data, and then coupled to AUTO within this new toolbox.

It is shown that the feasible region of a ground manoeuvre is defined by an algebraic constraint, where the desired turn-width forms a boundary. Bifurcation methods are used to follow this constraint as parameters, such as thrust and steering angle, are varied. From the practical point of view, size and location of the feasible region give a clear picture of whether or not a ground manoeuvre can be conducted. The bifurcation diagrams considered in this study encapsulate all the information that a design engineer would need in terms of turn-widths, edge-clearance distances, operating velocities, and steering angles. Therefore, bifurcation analysis provides an additional tool that can significantly enhance insight into the parameters that influence the performance of the aircraft on the ground, and so may contribute to a more mature product when flight testing commences.

II. Analysing Ground Manoeuvres

Aeroplane characteristics manuals from the original equipment manufacturers (OEM's) usually contain a baseline set of operating procedures that can be used to manoeuvre around airports. The three most common of these manoeuvres are the U-turn manoeuvre, an exit manoeuvre from the runway onto a taxiway, and the transition from one taxiway to another. There are several ways to conduct and interpret these manoeuvres due to the variability that is introduced by the pilot and the operating procedures of the airlines. The manuals also recognise this fact by stating that airline practices may vary to avoid excessive tyre wear and reduce possible maintenance problems. These variations from the standard procedures may arise due to physical constraints in the manoeuvring area, such as adverse gradients, limited area or a high risk of jet blast damage.

The purpose of this paper is not to study each of these manoeuvres in detail, but to show how bifurcation methods can be used in their analysis. To this end we focus on the U-turn manoeuvre, since there is still no clarity on how this manoeuvre is conducted, and also because it is one of the more challenging ground manoeuvres for pilots, especially in larger aircraft. FAA Code VI [7] requirements indicate that large aircraft such as the Airbus A380 and Boeing 747 are required to make a U-turn on a runway with a 60 metre width.

The following sections will break the U-turn manoeuvre into its constituent parts, which is followed by a description of three different methods for calculating each part. The first method uses a purely geometric approach, the second method is a simulation approach that uses the results from time-history simulations, and the third method is based on bifurcation analysis.

Validated models should always be used to serve as a reference point for the current state of the art. The baseline model used in this study is a validated A380 model that was developed in MSC.ADAMS within the Landing Gear group of Airbus, where it is mainly used for ground manoeuvrability studies. An equivalent model has also been implemented in SimMechanics [6], and it can be used for simulations or bifurcation analysis. Both MSC.ADAMS and SimMechanics are software packages that use the multibody-systems approach to dynamic behaviour. Within our MATLAB implementation, AUTO has direct access to the

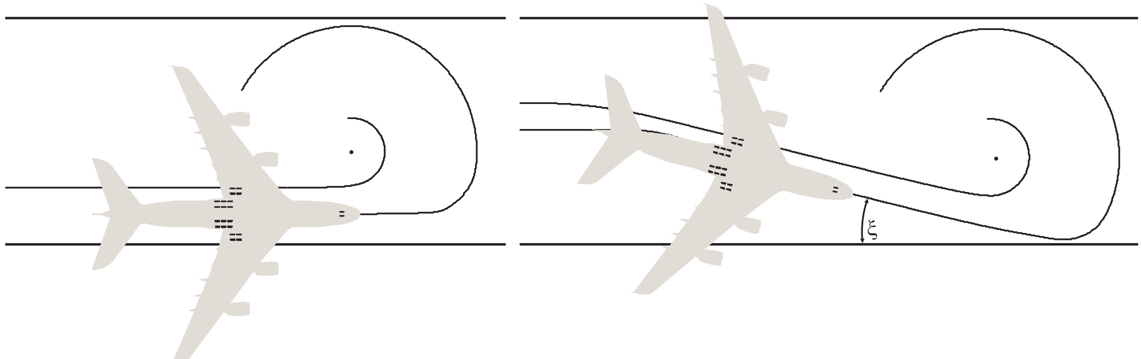


Figure 1: Edge of runway (EOR) and centre of runway (COR) U-turn manoeuvres.

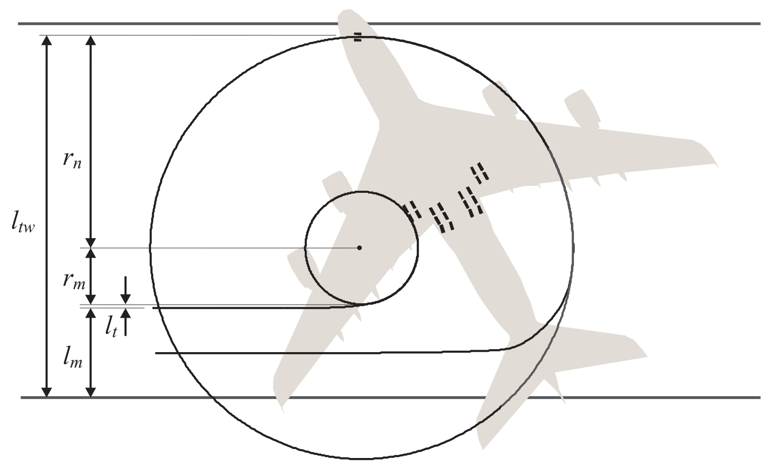


Figure 2: U-turn manoeuvre dimensions.

states of the SimMechanics model, even though the model equations are a black-box to the user. This is an important capability that allows one to integrate existing validated models with the bifurcation software, avoiding significant effort in redeveloping models for a specific application.

II.A. The U-turn Manoeuvre

A U-turn can be conducted in one of two ways. The first method is called an Edge-of-Runway (EOR) manoeuvre; it is conducted by placing the aircraft parallel to the side of the runway and then initiating the turn at any point. The second method is called the Centre-of-Runway (COR) manoeuvre; it is conducted by starting from the middle of the runway, traversing to the side of the runway at an angle, and then initiating the turn as soon as the nose gear reaches the edge of the runway. The COR method tends to allow for larger turn margins due to a shift in the centre of rotation towards the edge of the runway. Figure 1 depicts the two different approaches. Only the EOR method will be discussed, as the methods are essentially the same, apart from the initial starting points.

The steps for conducting the U-turn are:

1. To align the aircraft with the edge of the runway. The pilots will leave some space between the gears and the edge of the runway, but for the purposes of the simulations the outer plane of the outer wing gear tyres are aligned with the edge of the runway.

2. Set the aircraft in motion by applying thrust to all the engines.

3. Increase the engine thrust on the outboard engine while decreasing the thrust on the inboard engines. These two actions are done at the same time in the simulations, whereas it will most likely be done separately in reality.

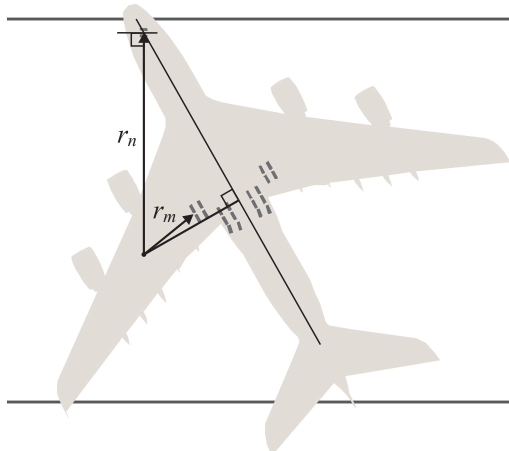


Figure 3: Geometric approach for finding the turn radius.

4. Apply the brake pedal on the side that the aircraft will turn into, meaning that the left-hand pedal is used for anti-clockwise turns, while the right-hand pedal is used for clockwise turns.

5. Set the nose gear steering angle by using the tiller.

It is worth noting that the above steps are implemented in this order for the purpose of the simulations, but these steps can in fact be performed in a different order or can be combined. It is proposed that the fourth and fifth steps are the most important, and will be similar for the EOR and COR manoeuvres. Therefore, only the EOR manoeuvre is analysed, from which the COR turn-width follows. Figure 2 contains the dimensions of importance for a U-turn, which feed into the turn-width l_{tw} given by

$$l_{tw} = l_m + l_t + r_m + r_n. \tag{1}$$

Here, the outer width between the outer most gears is represented by l_m, and l_t represents a transition distance. This transition distance is a function of the nose gear velocity, steering rate and final steering angle. It is assumed that the aircraft makes a steady turn, hence the radius does not change after the transition period. Two radii are thus of importance. The first is the radius of the nose gear r_n, while the second is the radius of the inner gear r_m. The reference point for the inner gear is however not located at the bearing point between the strut and the bogey beam, but a point that is offset by half an axle- and half a wheel-width from the bearing point, towards the centre of rotation. These radii can be altered by using different thrust and braking combinations.

The turn-width from the COR method can be calculated by subtracting a geometric distance from the EOR turn-width solution. The maximum additional distance l_{cor} that can be acquired by using the COR procedure can be obtained by

$$l_{cor} = (l_m + l_t + r_m)(1 - \cos(\xi)), \tag{2}$$

where ξ represents the angle that is formed between the fuselage longitudinal-axis and the edge of the runway. The optimum angle would be the angle that is formed between the fuselage longitudinal-axis and a line that is drawn between the nose gear and the outer main gear.

II.B. The Geometric Approach to the U-turn

The geometric method seems to be the most widely used method for calculating the turn radius of aircraft [8]. Automotive references go further by recognising that the radius of turn is dependent on the velocity of the vehicle, and consequently some assumptions are made with regards to the slip-angles that can be generated [9]. These slip-angles then contribute to the forces in the tyre and the resulting radius of turn.

The main assumption is however that the slip-angles remain small ($< 5°$), and consequently a lateral stiffness coefficient can be used. A formula can then be used to generate the turn radius. This approximation is however not sufficient for aircraft, as the tyres can operate at very large slip-angles (up to $90°$) where linear approximations cannot be made. The difficulty of obtaining an accurate dynamic formula might be the reason for the wide-spread use of the geometric method.

An example of the geometric method can be found in the paper on landing gear design by Chai and Mason [8], where it is proposed that the centre of rotation lies at the intersection between a line that is drawn perpendicular to the mean distance between the main gear posts, and a line extended from the nose gear axle. Figure 3 depicts this approach, which is generally used in the initial concept stages of an aircraft programme. The centre of rotation is highly dependent on different thrust and braking combinations, which are completely ignored within this geometric approach. Other information such as tyre forces are also not available. The final turn-width can then be calculated with equation (1), while assuming a constant factor for the transition distance l_t. Geometric methods might be sufficient for small aircraft with tricycle arrangements [10], but will not be sufficient for larger aircraft, especially those with more landing gears.

III. U-turn width from Simulations

A more reliable, yet expensive approach, is to conduct simulations, where the design space is divided into a grid of different combinations of steering angles and velocities. All the dynamic effects are taken into consideration, leading to more reliable values for the turn-width. The user is able to test detailed steering- and braking control-logic algorithms, balancing the desired turn-width against the loads on the gear and tyres. A penalty is however incurred due to the difficulty in automating the testing of such manoeuvres, as well as the high CPU times required for such simulations. This simulation approach is therefore best suited to detailed design studies in the later stages of a major aircraft programme. Simulations are nonetheless conducted at very specific operating conditions during the concept phase for trade-off studies.

III.A. SimMechanics Model

The basis for a reliable simulation is a validated model of the aircraft. The first step of the modelling is to describe the rigid parts and the joints connecting the parts [11], where a part is described by its mass, inertia and orientation. Specifically, in the model considered here, all 5 shock absorbers are constrained by cylindrical joints, where the nose gear rotation is driven by an angular motion, and the main gear rotations are constrained by torsional spring damper elements. The next step is the addition of internal force elements on the translational degrees of freedom of the joints. These forces are known as *line-of-sight* forces, and represent the shock absorbers and tyre forces. The A380 also has steering actuation on the aft axles of the body gears, which are represented by revolute joints with angular motions.

In the A380 SimMechanics model the tyres (2 nose and 20 main) are represented by impact functions that switch on as soon as the distance between the wheel centre and the tyre becomes less than the wheel radius. The other tyre forces are provided by means of lookup tables of empirical data. External forces such as thrust and aerodynamic forces are then added, and are known as *action-only-forces.* All geometric aspects are parameterised, from the axle widths, wheel dimensions, gear positions, to the rake angles on the gears. This means that all joint definitions and forces are automatically updated when the design parameters are changed.

Figure 4 shows the top-level Simulink model that contains the underlying A380 SimMechanics model. The model contains 38 states that describe the CG and gear positions and velocities. To provide for configuring the model, extensive use is made of the new object oriented features in MATLAB. This has added to the ease of use and robustness of the models - an ideal situation for industrial use. The user can enter one command that will configure the thrust, steering and braking configurations. The models can also be configured for towing operations. The reader is referred to a previous paper [3,4] to obtain exact details on how the models are constructed for use with bifurcation software.

III.A.1. Tyre Modelling

Apart from the aerodynamic, propulsive, and gravitational forces, all other loads on the aircraft are applied at the tyre-ground interface. Tubeless radial tyres are generally used for aircraft due to better failure characteristics when compared with bias-ply tyres [9]. The forces generated by the tyres have a dominant

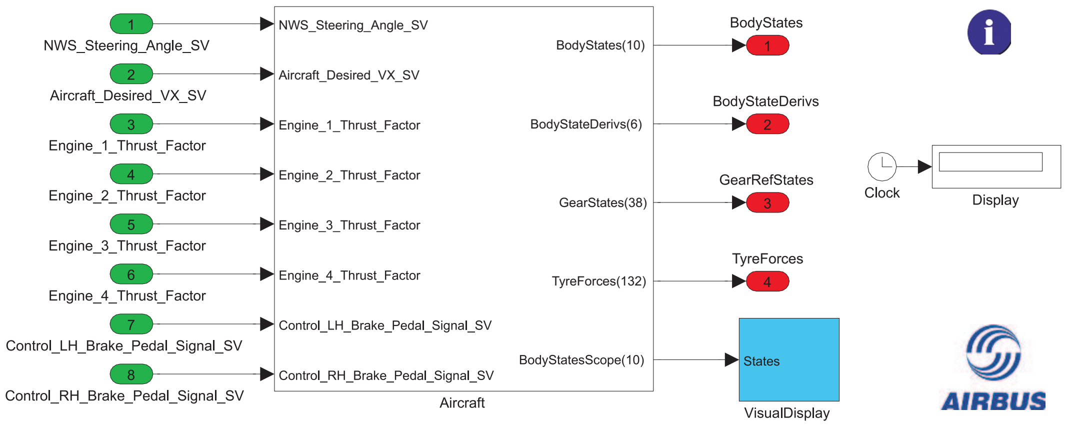
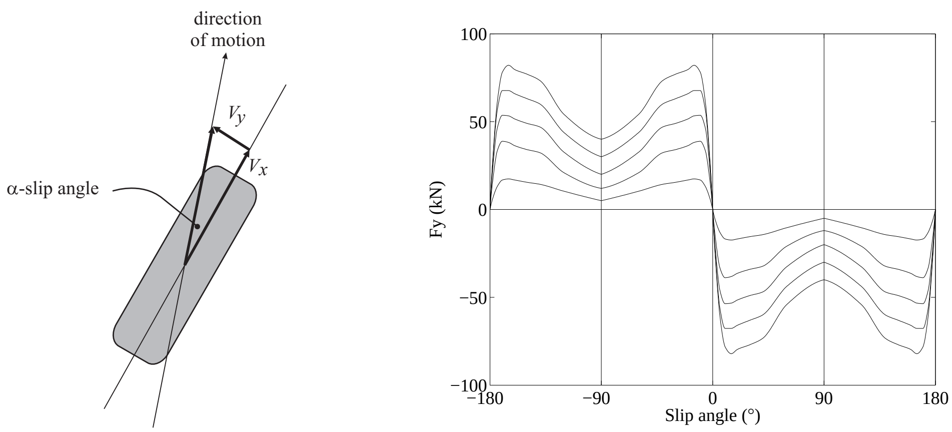


Figure 4: SimMechanics model.



(a) Slip-angle definition.

(b) Lateral tyre forces.

Figure 5: Relevant tyre quantities.

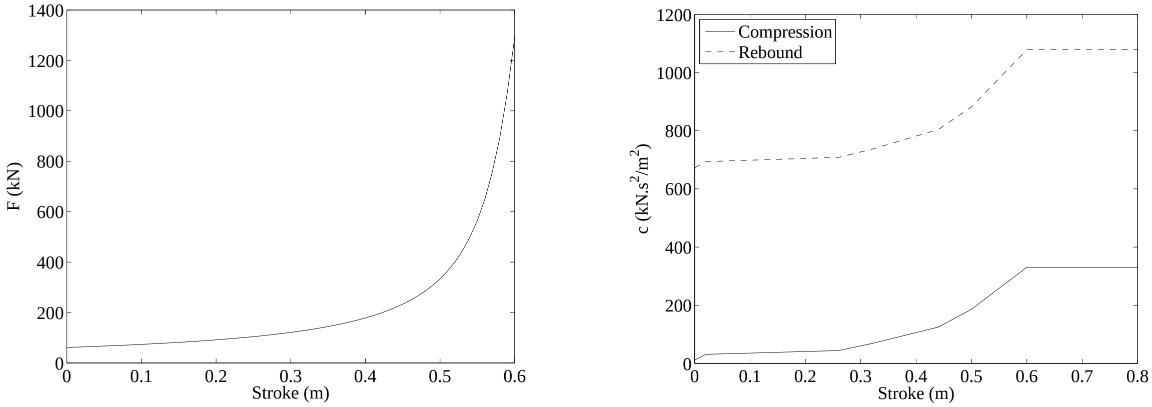
effect over the aerodynamic forces at low velocities. The vertical force component on the tyre can be approximated by a linear spring and damper system [3, 4, 11]. The total force is

$$F_z = -k_z\delta_z - c_z V_z = -k_z\delta_z - 2\zeta\sqrt{m_t k_z V_z}, \tag{3}$$

where V_z is the vertical velocity of the tyre, and δ_z is the tyre deflection representing the change in tyre diameter between the loaded and unloaded condition. The coefficients of equation (4) are determined from experiments, and are usually provided by the tyre manufacturers to the airframe OEM's. Several theories exist for the rolling resistance of a wheel, of which the following explanation seems the most plausible [11]. Rolling resistance on hard surfaces is caused by hysteresis in the rubber of the tyre, where the pressure in the leading half of the contact patch is higher than in the trailing half. A horizontal force in the opposite direction of the wheel movement is needed to maintain an equilibrium and is known as the *rolling resistance* [9]. The ratio of the rolling resistance F_x to vertical load F_z on the tyre is known as the *coefficient of rolling resistance* μ_R, where a value of 0.02 is typically used for aircraft tyres [12]. The model implements an adapted Coulomb friction model that is smoothed around the stationary point, as given by

$$F_x = -\mu_R F_z \tanh{(V_x)}. \tag{4}$$

Lateral motion is generated by directing the tyre at an angle to the direction of motion, leading to a lateral force. This angle, α, is known as the *slip-angle*; see figure 5(a). The relationship between the lateral force



(a) Stiffness

(b) Damping

Figure 6: Oleo properties.

and the slip-angle is linear for small slip-angles, and is usually defined by a cornering stiffness coefficient in the automotive industry [11], where maximum slip-angles of 5° seem to be the norm [11]. Slip-angles on aircraft often go up to 90° during normal manoeuvres, making it necessary to define the tyre properties over all possible slip-angles. Figure 5(b) contains the definition for the lateral force over the entire range of slip-angles.

Braking is implemented by adding a brake force term to the longitudinal force. The available lateral force that the tyre can generate is reduced when braking is applied, and is taken into account by using the concept of a *traction circle* [9]. The resultant force falls within the traction circle and reaches a maximum at the circle boundary.

III.A.2. Oleo Modelling

The main function of a shock absorber is to dissipate energy during landing and taxiing, so that the forces that are introduced into the airframe are within operating limits [10]. Large passenger aircraft tend to have oleo-pneumatic shock absorbers, due to the superior efficiency-to-weight benefit that these systems provide [10]. The gas in the upper chamber acts as a spring when it is compressed. A diaphragm or a piston can be used to separate the oil and the gas, otherwise they are left to mix. Energy dissipation takes place at the orifices, which act as the damping element of the shock absorber.

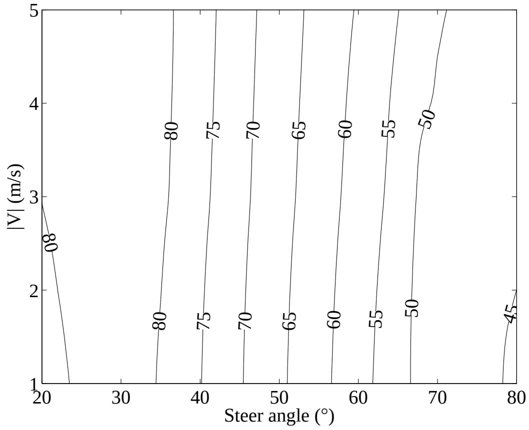
A level attitude is desired when the aircraft is standing on the runway, and therefore the static load should be calculated using the maximum aircraft weight, at the fore and aft CG positions. The extended stroke lengths are calculated from the aircraft geometric considerations, and then an initial estimate is made of the stroke that is required, based on previous aircraft. Compression ratios are then chosen based on experience, where a static to extended ratio of 4:1 and a compressed to static ratio of 3:1 are generally used [10]. The spring force F_k can then be calculated by multiplying the pressure inside the piston by the piston area. Figure 6(a) contains the spring curve that is used for the nose gear.

Damping is provided when the oil moves through the orifices within the orifice block and the recoil rings, where the damping force is dependent on the direction of motion. The damping force F_c is calculated from

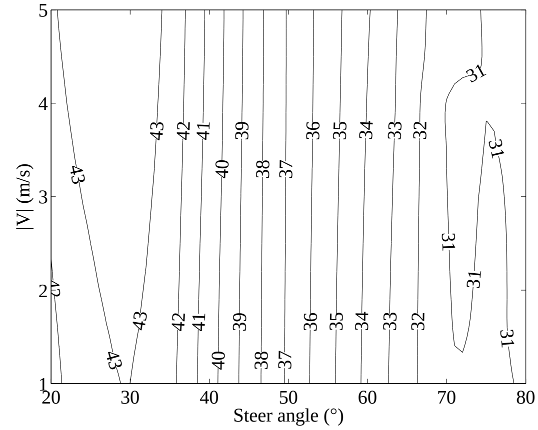
$$F_c = c(l_s){V_o}^2, \tag{5}$$

where the damping coefficient c is a function of the oleo stroke l_s. Figure 6(b) contains the damping coefficients for the nose gear. The combined force in the oleo F_o is then calculated as

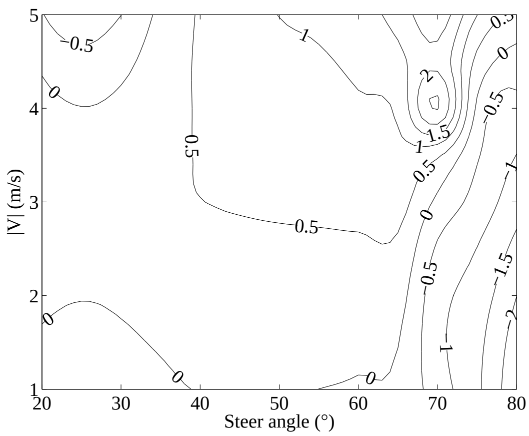
$$F_o = F_k(l_s) - F_c(l_s). \tag{6}$$



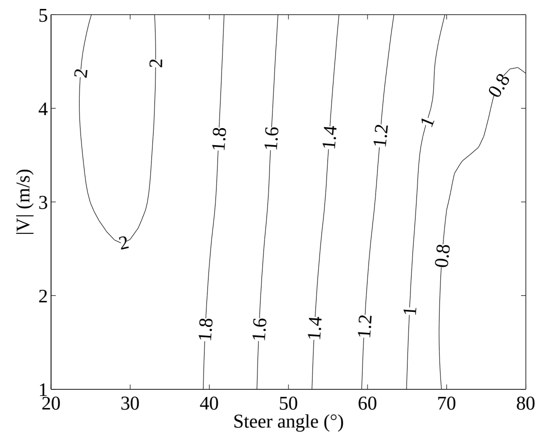
(a) Turn-width l_{tw} for medium weight aircraft.



(b) Turn radius of nose gear r_n.



(c) Transition distance l_t.



(d) Centre of runway additional margin l_{cor}.

Figure 7: U-turn performance from simulations.

III.B. U-turn Results from Simulations

We focus on the low-speed turn-width solutions over a range of fixed nose gear velocity magnitude levels ($1 - 5$ m/s). The velocity magnitude of the nose gear and CG will thus be similar when no steering input is given. In contrast, the CG velocity will drop as the steering angle is increased, while the nose gear velocity is being maintained. Asymmetric thrust configurations are of most interest, seeing that such configurations provide the smallest possible turn radii. A proportional-integral thrust controller is used on the outboard engine to ensure that the nose gear velocity is maintained, while the other engines are set to idle-thrust.

In reality the pilot will not set an exact velocity, but this is assumed for automation and comparison purposes. The gains of the controller are set in such a way as to represent the manner in which the pilot would operate the engine. The nose gear velocity will drop as the aircraft enters the turn, after which the pilot will increase the thrust to maintain the velocity. This thrust increase usually leads to an overshoot of the desired velocity. The pilot corrects the overshoot by decreasing the thrust. The rest of the steps with regards to how the manoeuvres are conducted are as explained in section II.A.

A range of steering angles ($20° - 80°$) and velocities ($1 - 5$ m/s) were used for a medium weight A380 configuration. The selection of test points is a balance between capturing the most important phenomena against simulation run-times. The simulation run-time for this example was approximately 2.5 hours on an Intel 1.8 GHz processor. Greater fidelity can be obtained by using more test points, but this would be at a significant cost to the run-times. The nonlinear nature of tyres does however mean that areas of rapid transition may occur between certain velocities and steering angles, and consequently the mesh would need to be refined in these areas.

The results for the turn-width simulations in SimMechanics are shown in figure 7(a), where the area to the right of the 60 m contour line indicates the feasible operating region for a U-turn. The turn-width increases slightly with an increase in velocity at a specific steering angle, but the simulations seem to indicate that the turn-width is not greatly influenced by the velocity for this specific case.

The transition distance is shown in figure 7(c) and is highly dependent on the actions of the pilot. Reaction times were obtained from flight test data, indicating that the steering input took place over the course of 5 seconds. The biggest transition distance occurs at a steering angle of 70° and a velocity of 4 m/s, and could indicate an area where a transition of loads between tyres takes place. This would have to be verified by studying the tyre forces from the simulations.

Section II.A shows that an additional turn margin can be obtained by using the COR instead of the EOR method. It can be computed by using equation (2), and this distance is shown in figure 7(d). If it is assumed that the 60 m contour line in figure 7(a) indicates the boundary of the feasible region, then figure 7(d) indicates that an additional margin of approximately 1.3 m can be obtained by using the COR method.

IV. U-turn Performance Using the Bifurcation Approach

Similar steps as for the simulation approach are followed, up to the point where the steering input is required. Instead of feeding the steering input into an ODE solver, the steering input is provided to the continuation software AUTO. The software is configured to ensure that the states remain within certain prescribed constraints as the steering angle is varied. This provides an immense amount of freedom in the design process, and allows one to follow any solution of interest in the relevant parameters. The user can for instance set a specific condition (algebraic constraint) on the tyre forces, and then follow this condition directly without having to do exhaustive simulations, obtaining the envelope for the prescribed condition. In this study the steering angle was varied while the derivatives of the states were set to zero, essentially following the trim solutions of the aircraft.

IV.A. Bifurcation Methods

Dynamical systems theory provides a methodology for studying systems of nonlinear ordinary differential equations (ODEs). A key method is that of bifurcation analysis, where one identifies different ways in which the dynamics of the system can change. In combination with the numerical technique of continuation, one can perform a nonlinear stability analysis by following solutions and detecting their stability changes (bifurcations). The bifurcations can then be followed in more parameters to identify regions in parameter space that correspond to different behaviour of the system. See, for example [13] and [14] as entry points to the literature.

To summarise some basic ideas consider an ODE model of the form

$$\dot{u} = f(u, \lambda). \tag{7}$$

where u is an n-dimensional state vector, λ a (multidimensional) control parameter, and f a sufficiently smooth (typically nonlinear) function. In terms of standard equations of motion for an aircraft on the ground, the state vector u contains the aircraft translational and rotational states, along with the translational states of the oleos. The control parameter consists of the steering angle, thrust, the position of the CG, and possibly other relevant parameters. Equilibrium solutions of (7), also known as trim conditions, satisfy

$$f(u_0, \lambda) = 0. \tag{8}$$

The implicit function (8) defines a solution locus of equilibria, which is a one-dimensional solution curve when a single parameter, such as the steering angle, is varied. The stability of the equilibria can be determined from the $(n \times n)$ Jacobian matrix Df of partial derivatives of the function f with respect to the state u. Continuation software, such as the package AUTO [2] used here, is able to follow curves of equilibria while monitoring their stability. See also [15] for an overview of the different software packages that are available. Changes of stability, that is bifurcations, are automatically detected and can then be followed in additional parameters. Similarly, periodic solutions can be followed and their stability changes detected. The continuation of suitable solution curves allows one to build up a comprehensive picture of the overall dynamics in a systematic way.

Typical bifurcations such as saddle-node (fold) and Hopf-bifurcations (onset of oscillations) can be found in engineering systems. In this paper we only encounter fold bifurcations due to the low velocity and thrust ranges of this study. Previous work on ground manoeuvring has indeed found oscillatory behaviour at higher velocity and thrust ranges [1, 3, 4]. Bifurcation analysis is now a standard and powerful tool that is being used extensively in engineering applications, and more recently for the analysis of landing gears and aircraft ground dynamics [1, 3–5].

IV.B. AUTO Integration

Bifurcation methods have not been readily adopted by the engineering community because the methods and tools available have thus far been developed and used mainly within an academic environment. The development of a Dynamical Systems Toolbox within the MATLAB environment is our attempt to consolidate previous efforts at the University of Bristol to create a user-friendly environment for engineers. Other efforts around the world to develop dynamical systems software in MATLAB exist, such as MATCONT [16], but it appears that this has not been widely adopted by the engineering community. We have thus tried to obtain the best of both worlds by integrating the existing Fortran AUTO code into MATLAB via mex-functions. This allows us to use the speed of a lower level language with the user-friendly interface of MATLAB.

Another important aspect of the toolbox is that engineering tools such as Simulink and SimMechanics can be integrated with the dynamical systems software. In this way, industrially tested models can be used directly in this environment, without the need for converting models to a format that can be used by AUTO. More specifically, AUTO has direct access to the states of the Simulink/SimMechanics model.

More widespread use of the Dynamical Systems Toolbox will be promoted by providing documentation and reference material that is easy to use, with concrete examples for the user. We have combined most of the user manual of AUTO with our own examples, and integrated this into the MATLAB help environment. The Dynamical Systems Toolbox therefore feels like any other toolbox that has been developed for MATLAB, where the user can select the toolbox from the menu, with the accompanying help and search functionality. We have also started to develop components with the new object oriented programming capability in MATLAB, and hopefully this will enhance the pace at which new applications will be developed in the future.

IV.C. U-turn Results from Bifurcation Approach

We propose a method that uses bifurcation analysis to determine relevant dynamic quantities in dependance on key parameters. This then feeds into a geometric model where we make some reasonable assumptions about the transient dynamics. More specifically, the bifurcation analysis provides the steady state solutions for the velocity magnitudes V_n and V_m at the nose and wing gear, as well as the yaw rate ω_z. The turn radius for the nose gear (and similarly for any other point on the aircraft) can then be calculated as

$$r_n = \frac{V_n}{\omega_z}. \tag{9}$$

Figure 8 depicts the most important dimensions in the calculation. The centre of rotation (x_o, y_o) in the left hand figure is not readily obtainable from the radii alone, but can be calculated if the directions of the velocity vectors (V_n and V_m) are known. This does however lead to unwieldy geometric calculations. An alternative approach is to draw the loci (which forms a circle) of the possible turn-centre solutions for each respective reference point. The outer intersection point relative to the fuselage centre line is then the solution for the turn-centre. The right hand figure shows how this method is implemented. The turn-width can thus be calculated by using equation (1), where accurate information is available for all the variables, apart from l_t, the transition distance. It is proposed that upper and lower bounds are chosen for this distance due to the variability of this value, which will then give the engineer a clear indication of what a best- and worst-case turn-width would be. The transition distance can be determined by simulations or from test data, but should be representative of the aircraft response due to normal pilot inputs.

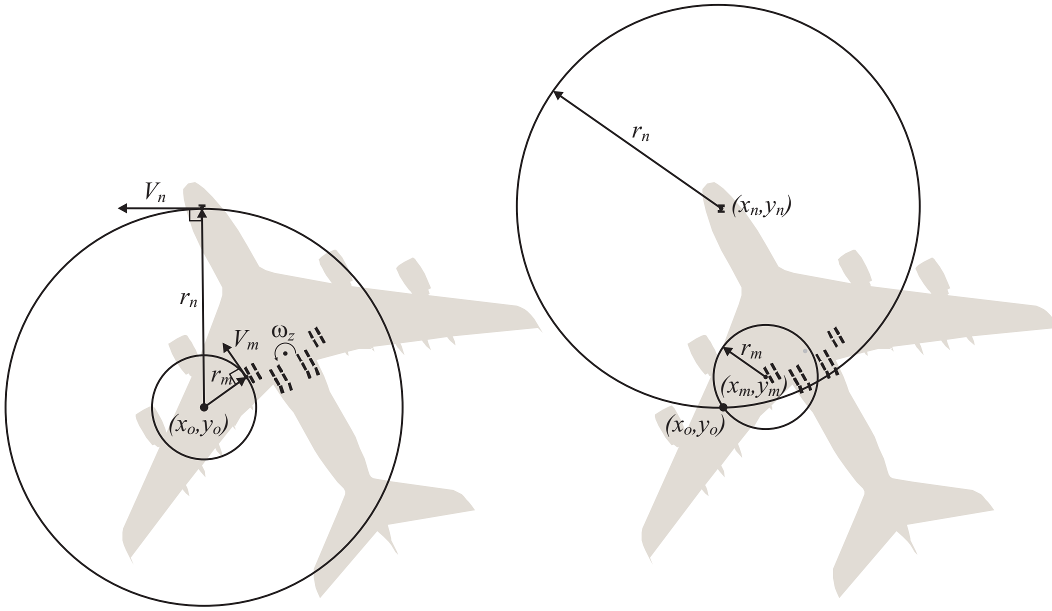


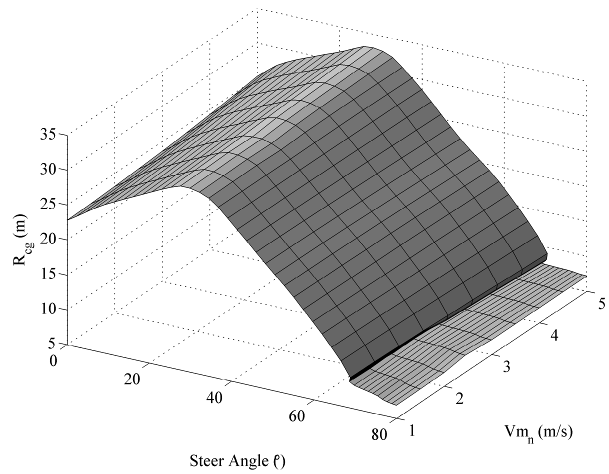
Figure 8: Turn radius and centre of rotation.

Figure 9(a) shows the turn radius for the CG, indicating that the turn radius initially increases and reaches a maximum at approximately 30°, after which it drops rapidly with an increase in steering angle. The initial increase can be attributed to the fact that the controller is trying to maintain a specific velocity at the nose gear, while the nose gear tyres have not reached their optimum lateral loading condition yet. This is not a realistic operational scenario, as the nose gear is essentially being dragged sideways due to the low steering angles. The pilot would in fact operate the nose gear at larger steering angles. The bifurcation diagram also shows two fold bifurcations at steer angles of approximately 65° and 68°, which indicates that a qualitative change in the dynamics takes place in this region. The aim of this paper is not to investigate this phenomenon, but it suggests that a new loading pattern emerges after the folds. This confirms our initial observations of a qualitative change in tyre dynamics from the simulation results. The run-time for obtaining the bifurcation diagrams was approximately 15 minutes on an Intel 1.8 GHz processor, which constitutes an order of magnitude increase in the analysis speed compared to that of simulations.

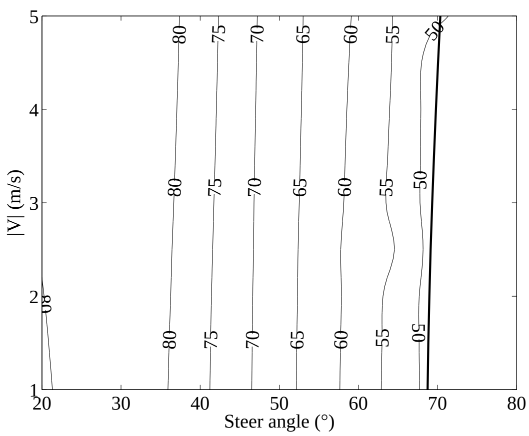
Equation (1) in section II.A was used to calculate the turn-width for this configuration, where we assumed a transition distance of 1 m. This leads to a conservative estimate at velocities below 4 m/s, the region where a pilot is expected to operate. The thick lines in figure 9(b) and 9(c) close to the 50 m contour indicate the location of the top fold. It can be seen that there is good agreement between the turn-width results from the simulation in figure 7(a) and those from the continuation analysis in figure 9(b). The turn radius of the nose gear is shown in figure 9(c), and it again shows very good agreement with the results from the simulations in figure 7(b).

IV.D. Turn-centre

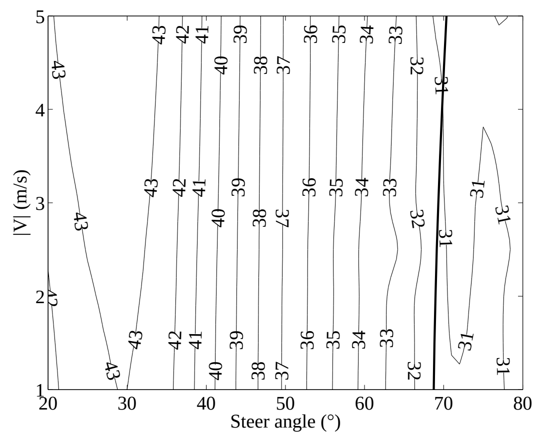
The position of the turn-centre is important as it can play a large part in the types of loads that can be generated in the gears. This is especially true for larger aircraft, where a pivot-turn (turn around the inner main gear) could for instance introduce large torsional loads on the inner main gear. Design engineers also overlay the aircraft and turn-centre positions onto airport drawings, to determine whether the aircraft can manoeuvre around specific corners. However, this geometric approach ignores the dynamic effects. Furthermore, simulations are conducted at specific airports to ensure airport compatibility, but is often quite complex to set up if many airports need to be considered. The bifurcation approach provides all the dynamic information that is needed for accurate calculations of the turn radii and turn-centre, and therefore it is proposed that this method can be used to obtain a more accurate estimate of ground performance (as long as the transient effects are understood). It is assumed that the aircraft will be able to conduct a safe turn if the calculated turn radius is smaller than that of the turn that needs to be negotiated. Bifurcation



(a) Bifurcation diagram for the turn radius of the CG.



(b) Turn-width l_{tw} .



(c) Turn radius of nose gear r_n.

Figure 9: U-turn performance from continuation analysis for medium weight aircraft.

diagrams also provide steering angles and velocities for safe operations and can be added to the interpretation of the results.

Figure 10 compares the geometric and the bifurcation approach. By construction the geometric turn-centre lies on a straight line perpendicular to the main aircraft axis. This longitudinal position is in good agreement with the calculated actual turn-centres. However, the lateral positions are substantially different for similar steering angles. Figure 10 clearly shows that the turn-centre prediction for low steering angles from the geometric methods are inaccurate for relatively small steering angles. Note that the calculations for the exact values were done here for an asymmetric thrust case with asymmetric braking, which cannot be included in the geometric calculations. The geometric predictions are closer to the correct values for higher steering angles, but they are still out by several metres. The longitudinal position at high steering angles does seem to be in good agreement with the position mentioned by Chai and Mason [8]. We can therefore conclude that it is imperative to calculate the actual turn radius as a function of the steering angle by other than purely geometric means. Bifurcation analysis emerges as a practical tool for this approach.

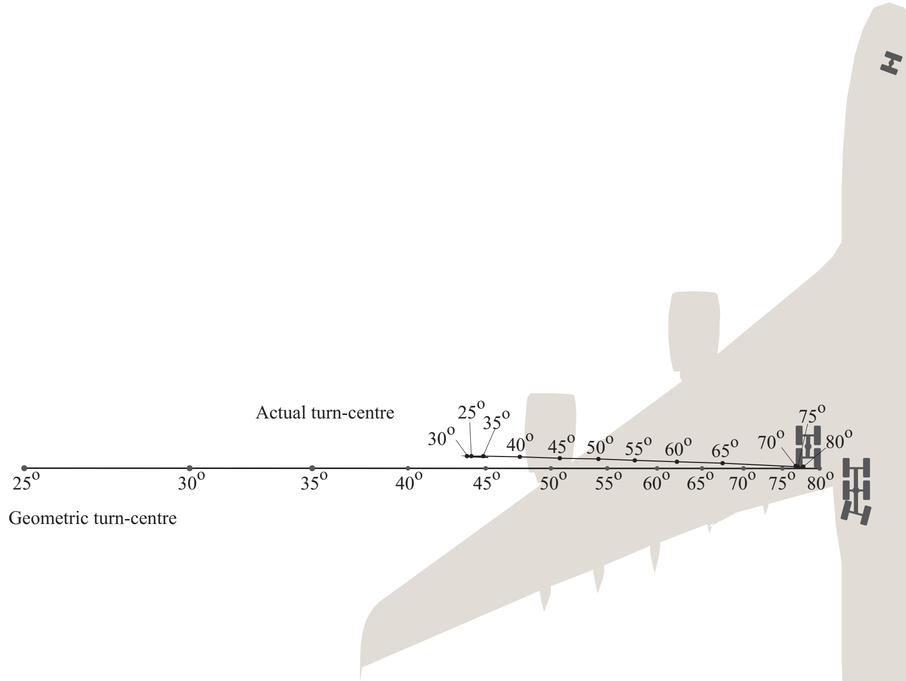


Figure 10: Position of turn-centre for steering angles from 25° to 80° calculated and geometric approaches.

V. Conclusions

We presented for the first time an in-depth analysis of the U-turn manoeuvre. A comparison was made between a widely used geometric method, a simulation-based approach, and a bifurcation analysis approach. The geometric method uses purely geometric and static relationships between the gear positions to determine the turn radius of the aircraft. It is a simple method to use, but engine thrust, tyre and brake inputs are ignored. Hence, the computed turn radii are generally not reliable as a result of the highly nonlinear nature of landing gear systems. Namely, small perturbations in velocity, steering angle or brake application may lead to significant differences in the final turn-width for the same basic geometry of wheel settings.

We showed how an industrially tested SimMechanics model is constructed and used for simulations of U-turn manoeuvres. A medium weight case was chosen with asymmetric thrust and braking inputs. The turn-width results from the simulations were presented as contour plots, and showed that the U-turn performance for the aircraft is well within the requirements for this particular configuration. We then used the same model to demonstrate how bifurcation analysis can be used to obtain turn-width results that are sufficiently close to that of the simulations. The advantage of the bifurcation analysis approach is that it is more efficient (in terms of run-times) and is also able to find qualitative changes in the dynamics, fold bifurcations in this case, that are not picked up by the simulations. Results are represented as bifurcation diagrams that also encapsulate all the information that a design engineer would need in terms of turn-widths, edge-clearance distances, operating velocities, and steering angles.

Overall, we conclude that the bifurcation analysis of ground manoeuvres would be suited to initial and detailed design studies. Future studies will compare the results of other mass, engine and braking configurations, and will also investigate the longitudinal movement of the turn-centre to determine the sensitivity of the turn-centre to parameters such as the CG position and tyre pressure. Investigations into the interpretation of the fold bifurcations will also be conducted. Bifurcation analysis can be used more generally for different types of ground manoeuvres. Towing studies will consider the effect of different tow-truck configurations, with the accompanying tyre loads for configurations with a towbar and with a towbarless truck. A longer term aim is to add the aerodynamics for high-speed stability investigations.

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